Code No: R21043 **R10** SET - 1

II B. Tech I Semester Regular Examinations, March – 2014
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Give the definition and Axioms of probability.

   b) Using Venn diagrams prove the Demorgan's laws:

   i) $(\overline{A \cup B}) = \overline{A} \cap \overline{B}$ ii) $(\overline{A} \cap \overline{B}) = \overline{A} \cap \overline{B})$ (7M+8M)

2. a) If the function $G_x(x) = K \sum_{n=1}^{N} n^3 u(x-n)$ to be a valid distribution function, find the value of 'K'.

   b) State and prove any four properties of probability density function. (7M+8M)

3. a) Find the skew for Gaussian distributed random variable.

   b) Explain about the monotonic transformations for a continuous random variable. (7M+8M)

4. a) State and prove central limit theorem for equal distributions.

   b) The joint density function of random variables X and Y is

   $f_{xy}(x, y) = \frac{1}{a} e^{-|x|-|y|}$, $-\infty < x < \infty$, $-\infty < y < \infty$.

   i) Are X and Y statistically independent variables.

   ii) Calculate the probability of $x \leq 1$ and $y \leq 0$. (7M+8M)
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5. a) Three random variables X_1, X_2, and X_3 represent samples of random noise voltage taken at three times. Their covariance matrix is defined by

$$[C_x] = \begin{bmatrix} 3.0 & 1.8 & 1.1 \\ 1.8 & 3.0 & 1.8 \\ 1.1 & 1.8 & 3.0 \end{bmatrix}$$

The transformation matrix

$$[T] = \begin{bmatrix} 4 & -1 & -2 \\ 2 & 2 & 1 \\ -3 & -1 & 3 \end{bmatrix}$$

Convert the variable to new random variables Y_1, Y_2 and Y_3. Find the covariance matrix of the new random variables.

b) State and prove any two properties of joint characteristic function. (8M+7M)

6. a) Consider a random process $X(t) = \cos(\omega t + \theta)$ where ω is a real constant and θ is a uniform random variable in $(0, \frac{\pi}{2})$. Find the average power in the process.

b) Derive the condition for a random process to be mean Ergodic. (8M+7M)

7. a) State and prove any three properties of Cross correlation function.

b) Derive the relation between Auto Correlation Function and PSD. (7M+8M)

8. a) Derive the relation between PSD of input & Cross PSD of input and output.

b) A WSS process X(t) has $R_{xx}(\tau) = Ae^{-a|\tau|}$ where A and 'a' are real constants is applied to input of LTI system with $h(t) = e^{-bt} u(t)$, where 'b' is a real positive constant. Find the PSD of the output of system. (7M+8M)

Code No: R21043 **R10** SET - 2

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(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
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1. a) State the following and explain
   i) Baye's Theorem ii) Conditional probability.
   b) What is the probability of picking an ace and a king from a 52 card deck? (9M+6M)

2. a) For a real constant b>0, c>0 and any 'a' find the condition on constant 'a' such that
   $$f_X(x) = \begin{cases} \left[1 - \frac{x}{b}\right], & 0 \le x \le c \\ 0 & elsewhere \end{cases}$$ is a valid pdf.
   b) State and explain the properties of conditional density function. (8M+7M)

3. a) Find the mean and variance of 'X + a', in terms of mean and variance of 'X'.
   b) Derive the relation between moment generating function and moments. (7M+8M)

4. a) Let X and Y are two independent random variables with
   $f_x(x) = \alpha e^{-\beta x} u(x)$ and
   $f_y(y) = \beta e^{-\beta y} u(y)$
   Find the density function of Z = X + Y for i) $\alpha \neq \beta$ ii) $\alpha = \beta$
   b) Write the properties of Joint distribution. (8M+7M)

5. Zero mean Gaussian random variables X_1, X_2 and X_3 having covariance matrix.
   $$[C_x] = \begin{bmatrix} 4 & 2.05 & 1.05 \\ 2.05 & 4 & 2.05 \\ 1.05 & 2.05 & 4 \end{bmatrix}$$
   Are transformed to new random variable Y_1, Y_2, Y_3.
   i) Find the covariance matrix of Y_1, Y_2 and Y_3.
   ii) Write expression for joint density function of Y_1, Y_2 and Y_3. (15 M)
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6. a) A random process $X(t) = A \cos(w_c t + \theta)$ where θ is a random variable uniformly distributed in the range $(0, 2\pi)$. Show that the process is ergodic in mean and correlation sense.
   b) Define covariance function and explain its properties. (8M+7M)

7. a) If the Auto Correlation Function of a WSS process is $R(\tau) = Ke^{-k|\tau|}$. Find its PSD.
   b) Check whether the following functions are valid PSDS or not. (8M+7M)
   i) $\frac{w^2}{w^6 + 3w^2 + 3}$ ii) $\frac{w^2}{w^2 + 16}$

8. a) Compute the overall Noise figure of a four stage cascaded system with following data:
   $F_1 = 10$, $F_2 = 5$, $F_3 = 8$, $F_4 = 12$
   $ga_1 = 50$, $ga_2 = 20$ and $ga_3 = 10$.
   b) State and prove any three properties of Narrow band Noise processes. (8M+7M)

Code No: R21043 **R10** SET - 3

II B. Tech I Semester Regular Examinations, March – 2014
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Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
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1. a) Using Venn diagram and proof, prove that
   $P(A \bigcup B/C) = P(A/C) + P(B/C) - P(A \bigcap B/C)$.
   b) Define probability in terms of relative frequency.
   c) Explain independent events. (7M+3M+5M)

2. a) A random variable X is Gaussian with mean $m_x = 0$ and $\sigma_x = 1$.
   i) What is the probability that |X|>2. ii) What is the probability that X>2.
   b) Draw the pdf of Rayleigh density function by giving its expression and find the value and X where it is maximum. (8M+7M)

3. a) Let X be a random variable which can take values 1, 2, 3 with probabilities $\frac{1}{3}, \frac{1}{6}$ *and* $\frac{1}{2}$ respectively. Find the 3^{rd} moment about the mean.
   b) If X is the number scored in a throw of a fair die, show that Chebyshev's inequality gives P{|x-m|>2.5}<0.4, where 'm' is mean of X, while actual probability is zero. (7M+8M)

4. a) The joint density function of three random variables X, Y and Z is
   $f_{xyz}(x, y, z) = 24xy^2z^3$, 0<x<1, 0<y<1, 0<z<1
   = 0, otherwise.
   i) Find the maginal densities $f_x(x)$, $f_y(y)$ and $f_z(z)$. ii) Find P(X>1/2, y<2, z>1/2)
   b) State and prove any four properties of joint density function. (8M+7M)

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5. For the joint characteristic function

$$Q_{xy}(w_1, w_2) = \exp\left[-\frac{1}{2}\left[\sigma_x^2 w_1^2 + 2\rho\sigma_x\sigma_y w_1 w_2 + \sigma_y^2 w_2^2\right]\right]$$

Find the Marginal characteristic functions of X and Y. (15M)

6. a) Consider a random process X(t) = 10cos (100t + φ) where φ is uniformly distributed random variable in the internal (-π,π). Show that the process is correlation ergodic.

b) State and prove any four properties of Auto Correlation Function. (7M+8M)

7. a) Derive the relation between PSD of x(t) and PSD of $\frac{dx(t)}{dt}$.

b) For a random process X(t) = Acos (wt + θ)+B sinwt where A and B are two uncorrelated random variables with zero mean and equal variances and w is a real constant. Find the ACF of X(t) and hence its PSD. (7M+8M)

8. a) Derive the relation between input and output ACF of an LTI system with impulse response h(t).

b) An amplifier with g_a = 40 dB and B_N = 20 kHz is found to have $T_0 = 10^0$ K. Find T_e and Noise figure. (8M+7M)

Code No: R21043 **R10** SET - 4

II B. Tech I Semester Regular Examinations, March – 2014
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(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
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1. a) State and prove Baye's Theorem.
   b) If A and B are two mutually exclusive events show that
   i) $P(A/B) = \dfrac{P(A)}{1 - P(B)}$ ii) $P(A/AUB) = \dfrac{P(A)}{P(A)+P(B)} if P(AUB) \neq 0$ (7M+8M)

2. a) Find the constant 'b' such that
   $f_x(x) = \begin{cases} \dfrac{e^{3x}}{4}, & 0 \le x \le b \\ 0, & elsewhere \end{cases}$ Is a valid density function.
   b) State and prove any four properties of CDF. (7M+8M)

3. a) If X has density function
   $f_x(x) = \exp(-x), \quad x > 0$
   $= 0, \quad x \le 0$
   Find the density function of $Y = X^2$
   b) Find the mean of a Gaussian distribution. (8M+7M)

4. a) State and prove the central limit theorem.
   b) If X and Y are two Gaussian random variables with zero mean find the pdf of a new random variable Z = X+Y. (7M+8M)

5. a) State and explain the properties of jointly Gaussian random variables.
   b) Random variables X and Y has joint density.
   $f_{xy}(x,y) = \frac{8}{3}u(x-2)u(y-1)xy^2\exp(4-2xy)$ undergo a transformation
   $T = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ to generate new random variables Y_1 and Y_2. Find joint density of Y_1 and Y_2.
   (7M+8M)

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6. a) Consider a random process X(t) = A coswt where ‘w’ is a constant and A is uniformly distributed over (0, 1). Find the ACF and Auto covariance of X(t).
   b) Explain mean Ergodic processes in brief. (8M+7M)

7. a) The PSD of a random process is S_{xx} (w) = $\begin{cases} \pi, & |w| < 1 \\ 0, & otherwise \end{cases}$. Find its ACF.
   b) State and prove any three properties of Power Spectral Density. (8M+7M)

8. a) A random process X(t) has ACF $R_{xx}(\tau) = A^2 + Be^{-|\tau|}$ where A, B are positive constants. Find the mean value of the system having impulse response

   h(t) = $\begin{cases} e^{-wt}, & t > 0 \\ 0, & t < 0 \end{cases}$

   b) Derive the equation for Noise figure of Cascaded system in terms of individual Noise figures (8M+7M)